Boundary layer high order concentration statistics

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evaluated by a Lagrangian model (Luhar et al., 2000)

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- ✓ extending the single-particle model to the movement of the centroid;
- ✓ parametrizing the dispersion around the centre of the cloud.

Lagrangian model for the barycenter

Langevin equation

$$dx_m = U(z_m)dt$$
$$dw_m = a_m(t, w_m, z_m)dt + b_m(t, z_m)dW(t)$$
$$dz_m = w_m dt$$

Fokker-Planck

$$\frac{\partial P_m}{\partial t} + w\frac{\partial P_m}{\partial z_m} = -\frac{\partial\left[a(t, w_m, z_m)P_m)\right]}{\partial w_m} + \frac{\sigma_m^2}{T_m}\frac{\partial^2 P_m}{\partial w_m^2}$$

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single-particle Lagrangian Time Scale

Quadratic assumption

$$a(w_m, z_m, t) = \alpha_m(z_m, t)w_m^2 + \beta_m(z_m, t)w_m + \gamma_m(z_m, t)$$

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Lagrangian model for the barycenter

$$\frac{\partial}{\partial t}\langle w^m \rangle + \alpha_z \langle w^{m+1} \rangle + \beta_z \langle w^m \rangle + \gamma_z \langle w^{m-1} \rangle = \frac{1}{m} \frac{\partial \langle w^{m+1} \rangle}{\partial z} - 2(m-1) \frac{\sigma_m^2}{T_m} \langle w^{m-2} \rangle$$

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Centroid Vertical Acceleration Parameters:

$$\alpha(z_m) = \frac{(1/3)(\partial\langle w_m^3\rangle/\partial t + \langle w_m^4\rangle/\partial z_m)}{\langle w_m^4\rangle - \langle w_m^3\rangle^2/\langle w_m^2\rangle - \langle w_m^2\rangle^2} - \frac{\langle w_m^3\rangle/(2\langle w_m^2\rangle)\left[\partial\langle w_m^3\rangle/\partial z_m - 2\langle w_m^2\rangle/T_m\right] + \langle w_m^2\rangle\partial\langle w_m^2\rangle/\partial z_m}{\langle w_m^4\rangle - \langle w_m^3\rangle^2/\langle w_m^2\rangle - \langle w_m^2\rangle^2}$$

$$\beta_m(z_m) = \frac{1}{2\langle w_m^2\rangle}\left[\frac{\partial\langle w_m^2\rangle}{\partial t} + \frac{\partial\langle w_m^3\rangle}{\partial z_m} - 2\langle w_m^3\rangle\alpha_m(z_m)\right] - \frac{1}{T_m}$$

$$\gamma_m(z_m) = \frac{\partial\langle w_m^2\rangle}{\partial z_m} - \langle w^2\rangle\alpha_m(z_m)$$

Energy Filter and Partition of Energy

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Filter Function

$$\langle w_m^n \rangle = \langle w^n \rangle \left[1 - \left(\frac{d^2}{d^2 + z_i^2} \right)^{\frac{1}{3}} \right]^{n/2}$$

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$$t_s = \left[\sigma_0^2 / (g\varepsilon) \right]^{1/3}$$

Relative Concentration PDF

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Parametrization of the relative concentration

$$p_{cr}(c|x, z, z_m) = \frac{\lambda^\lambda}{\langle c_r \rangle \Gamma(\lambda)} \left(\frac{c}{\langle c_r \rangle} \right)^{\lambda - 1} e^{-\frac{\lambda c}{\langle c_r \rangle}}$$

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$$\lambda = 1/i_{cr}^2$$

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Relative vertical position PDF

Gaussian Parametrisation

$$p_{zr}(x,z,z_m) = \frac{1}{\sqrt{2\pi}\sigma_{zr}} \sum_{n=-N}^{N} \left[e^{-\frac{(z-z_m+2nz_i)^2}{2\sigma_{zr}^2}} + e^{-\frac{(-z-z_m+2nz_i)^2}{2\sigma_{zr}^2}} \right]$$

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Skewed Parametrisation

$$p_{zr}(x,z,z_m) = \sum_{j=1}^{2} \sum_{n=-N}^{N} \frac{a_j}{\sqrt{2\pi}\sigma_j} \left[e^{-\frac{(z-z_m+2nz_i-\bar{z}_j)^2}{2\sigma_j^2}} + \right.$$

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$$S_{zr} = \frac{\langle (z-\langle z\rangle)^3\rangle - \langle (z_m-\langle z_m\rangle)^3\rangle}{\sigma_{zr}^3}$$

Case studies:

CBL

Water tank dispersion experiments: Willis and Deardorff (1976, 1978, 1981).

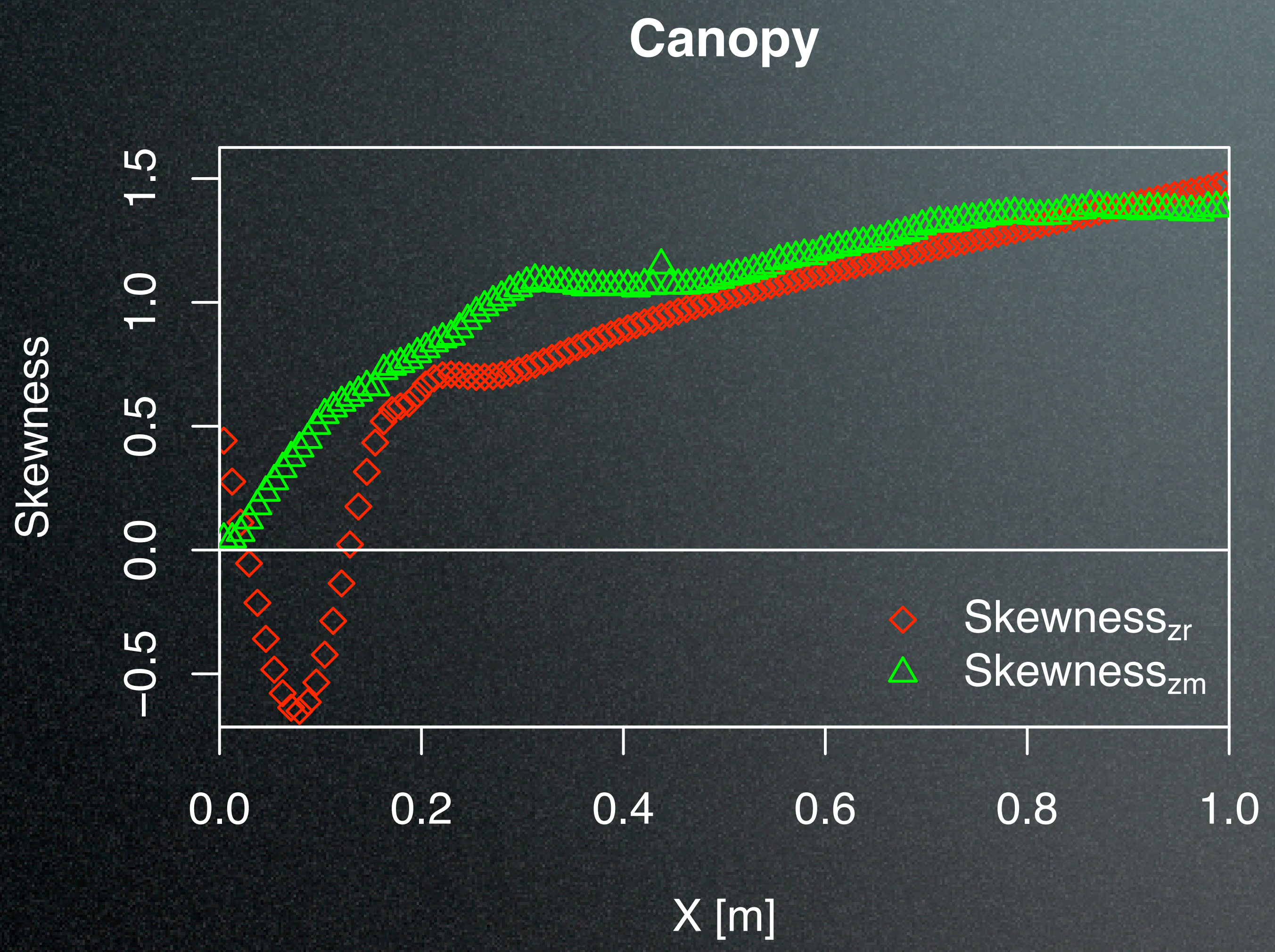
The turbulence used as imput of the Lagrangian stochastic model is decribed in Franzese et al. (1999) and Franzese (2003).

Canopy

Simulated vegetal canopy from wind tunnel experiments: Raupach et al. (1986), Legg et al. (1986), Coppin et al. (1986).

The turbulence used as input for the Lagrangian stochastic model was derived by polynomial and spline fit of the experimental data of Raupach et al. (1986), Legg et al. (1986), Cassiani et al. (2007).

Skewness of the single particle vertical position relative to the barycenter



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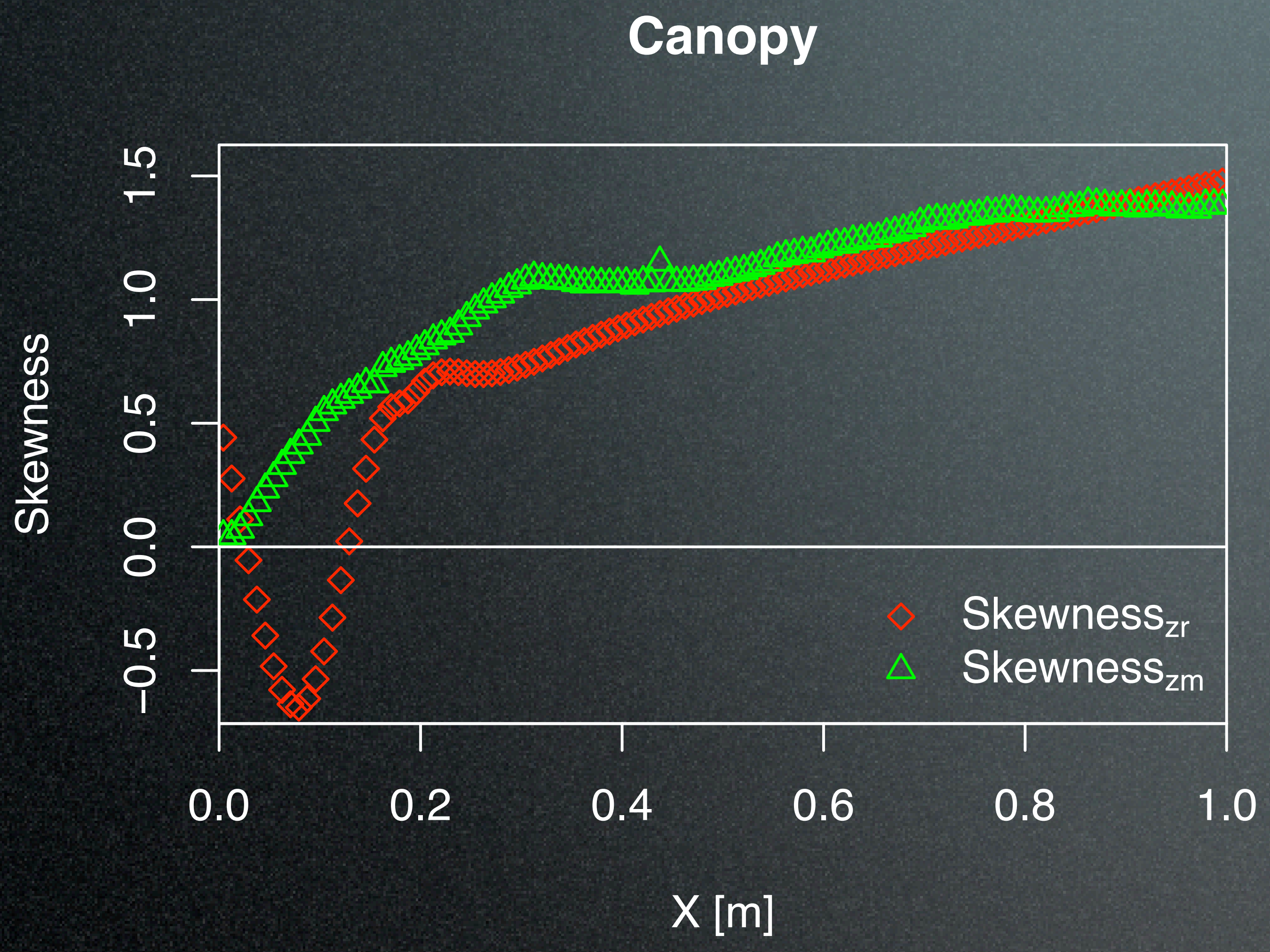
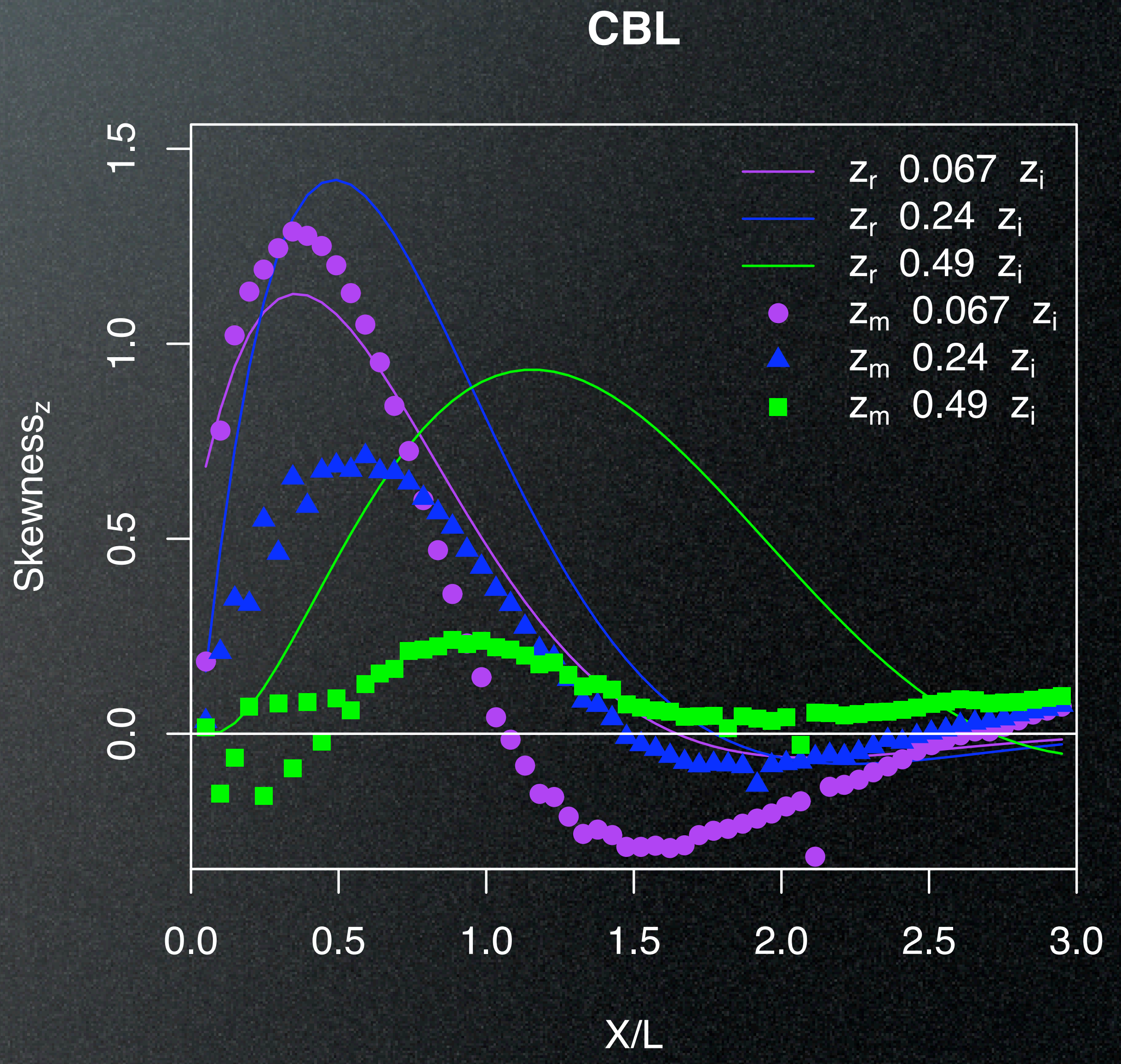
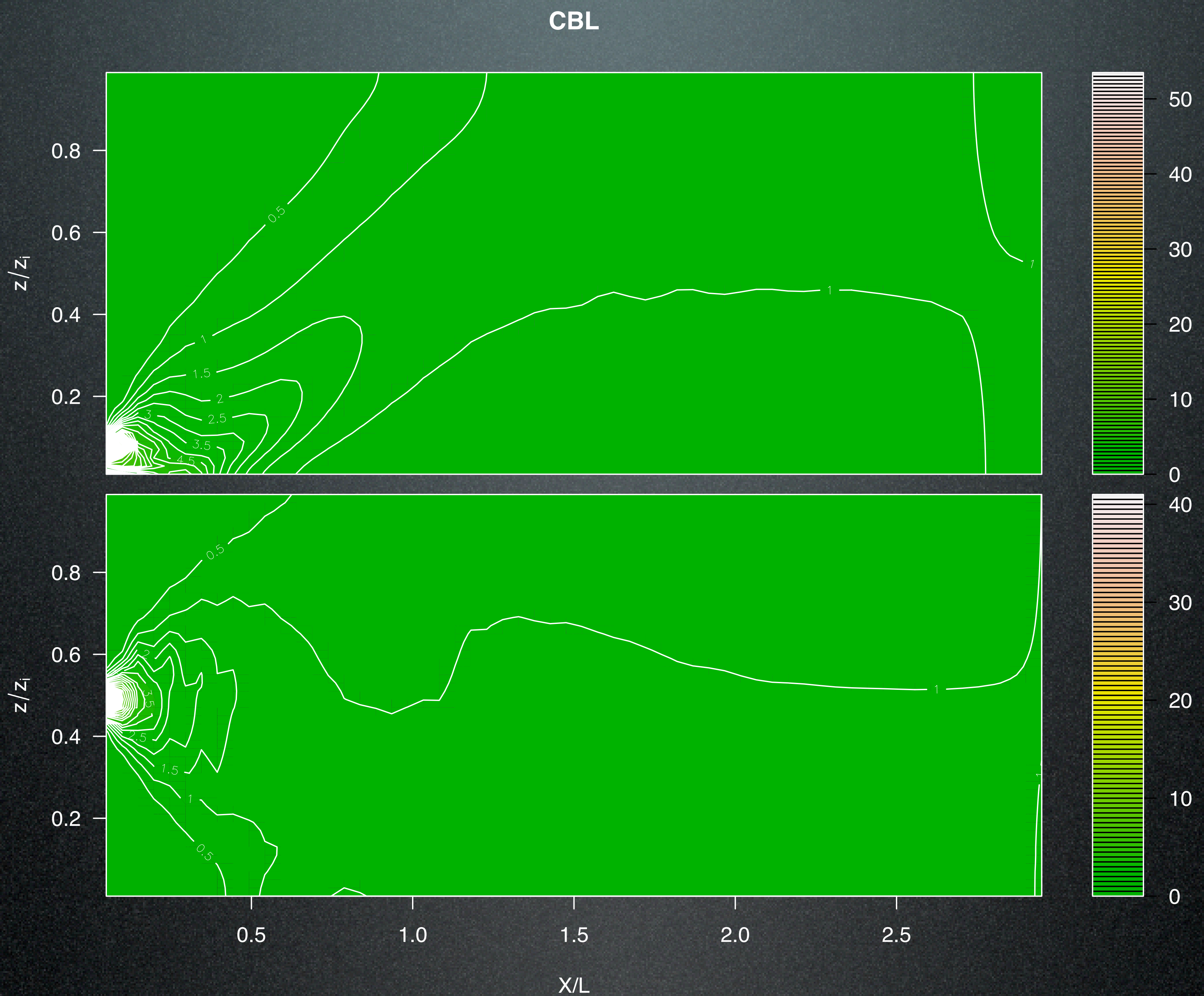




Mean Concentration Field - CBL



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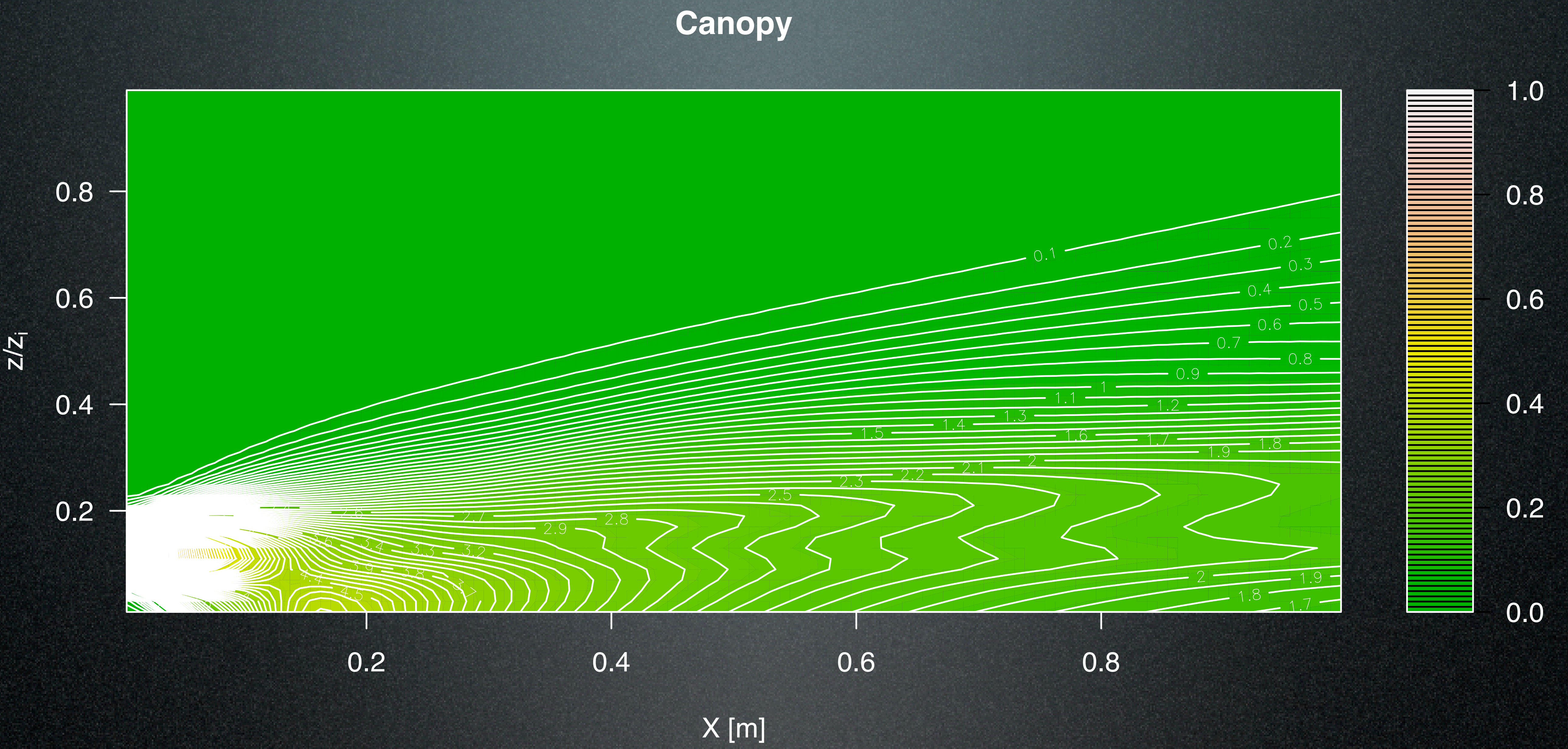

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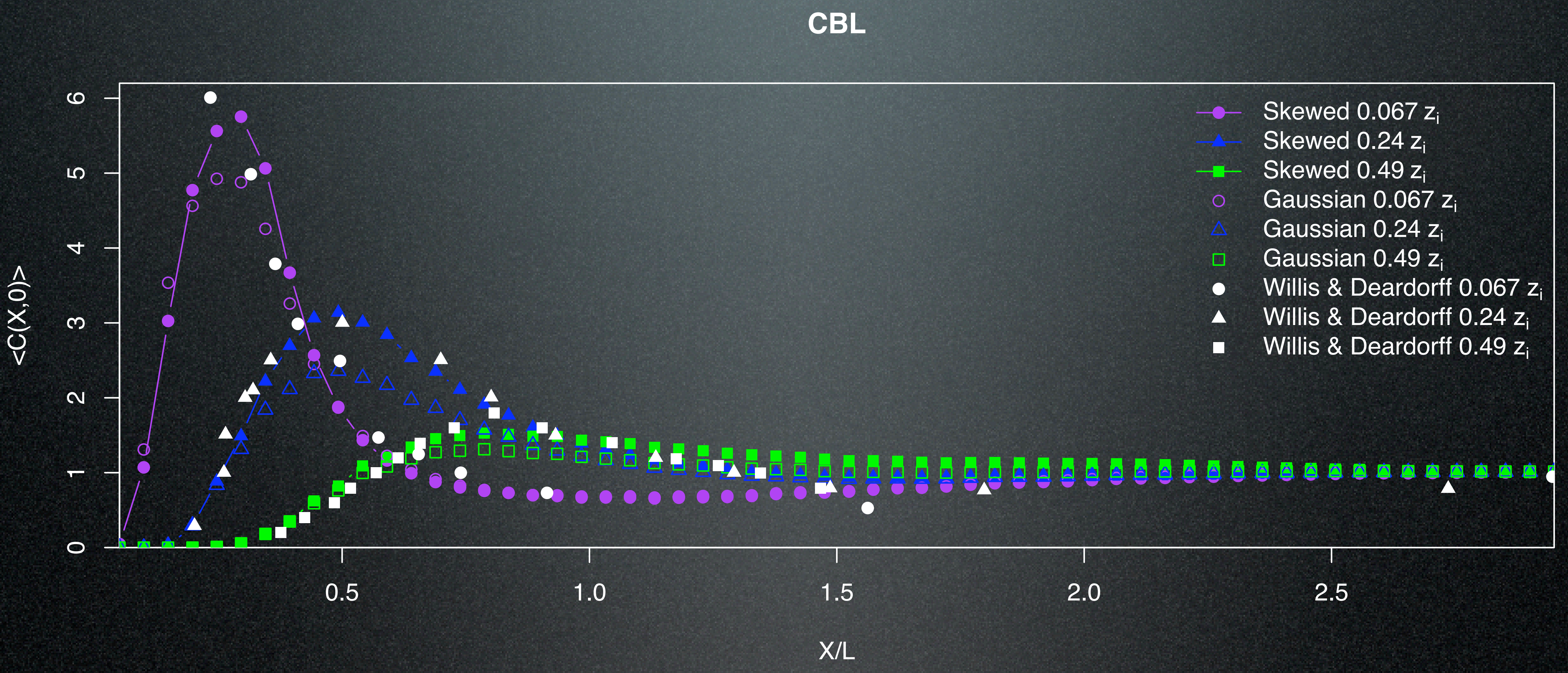
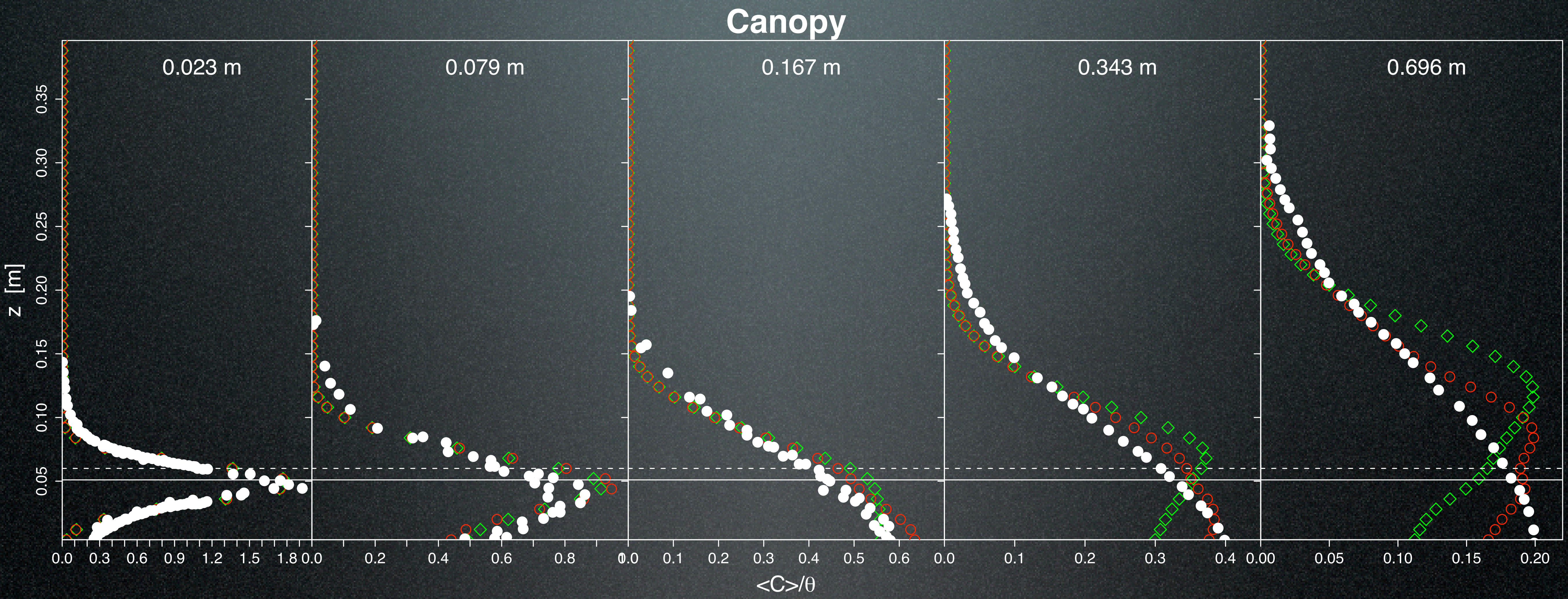
Ground Level Mean Concentration - CBL



Mean Concentration Profiles- Canopy

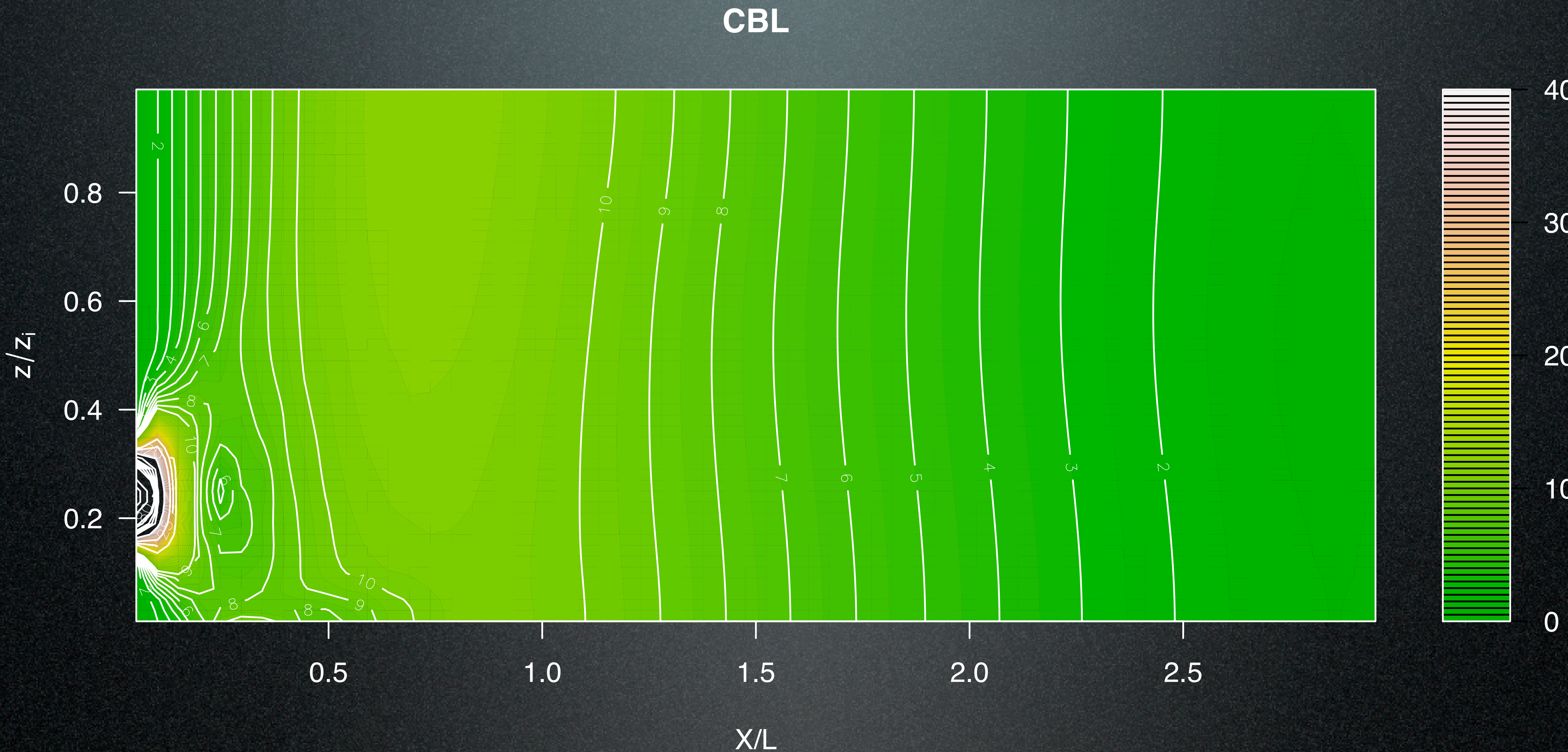


Legg et al. (1986) data

Fluctuating Plume Model, Gaussian Formulation

Fluctuating Plume Model, Skewed Formulation

Concentration Fluctuations' Field - CBL



$$\langle c^2(x,z)\rangle = i_{cr}^4 \frac{\Gamma(1/i_{cr}^2+2)}{\Gamma(1/i_{cr}^2)} \left(\frac{Q}{U}\right)^2 \int_0^{z_i} p_{zr}^2(x,z,z_m)\, p_m(x,z_m)\, dz_m$$

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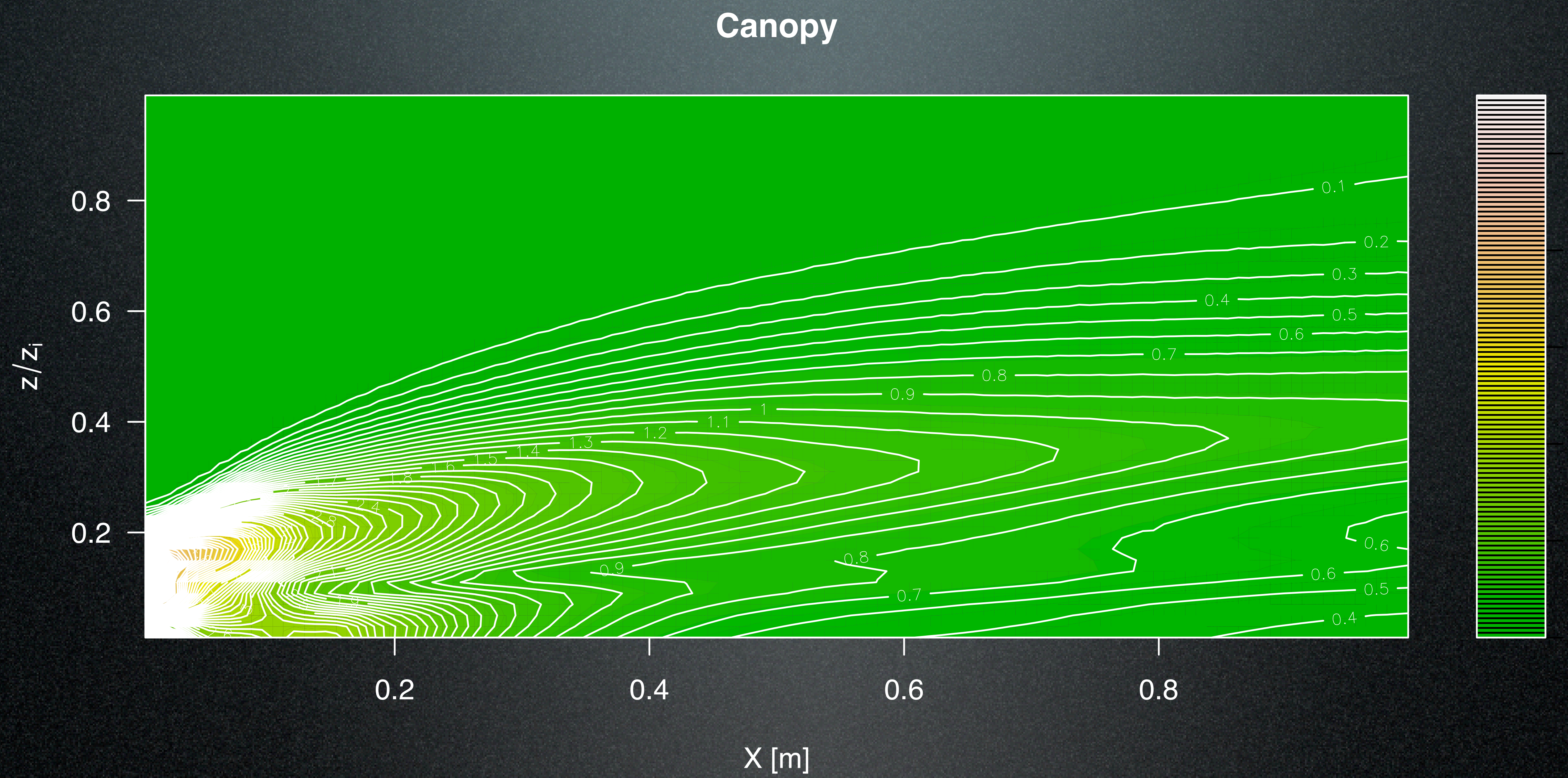

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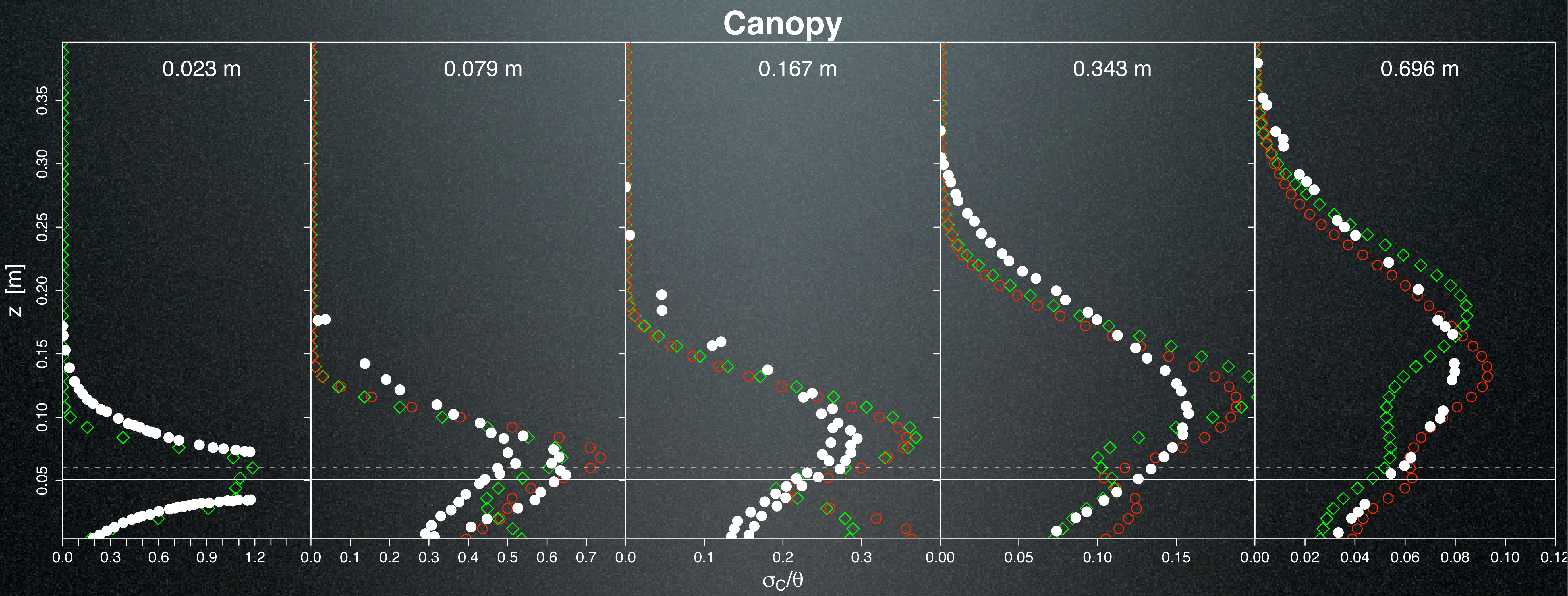

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High Order Concentration Statistics- Canopy

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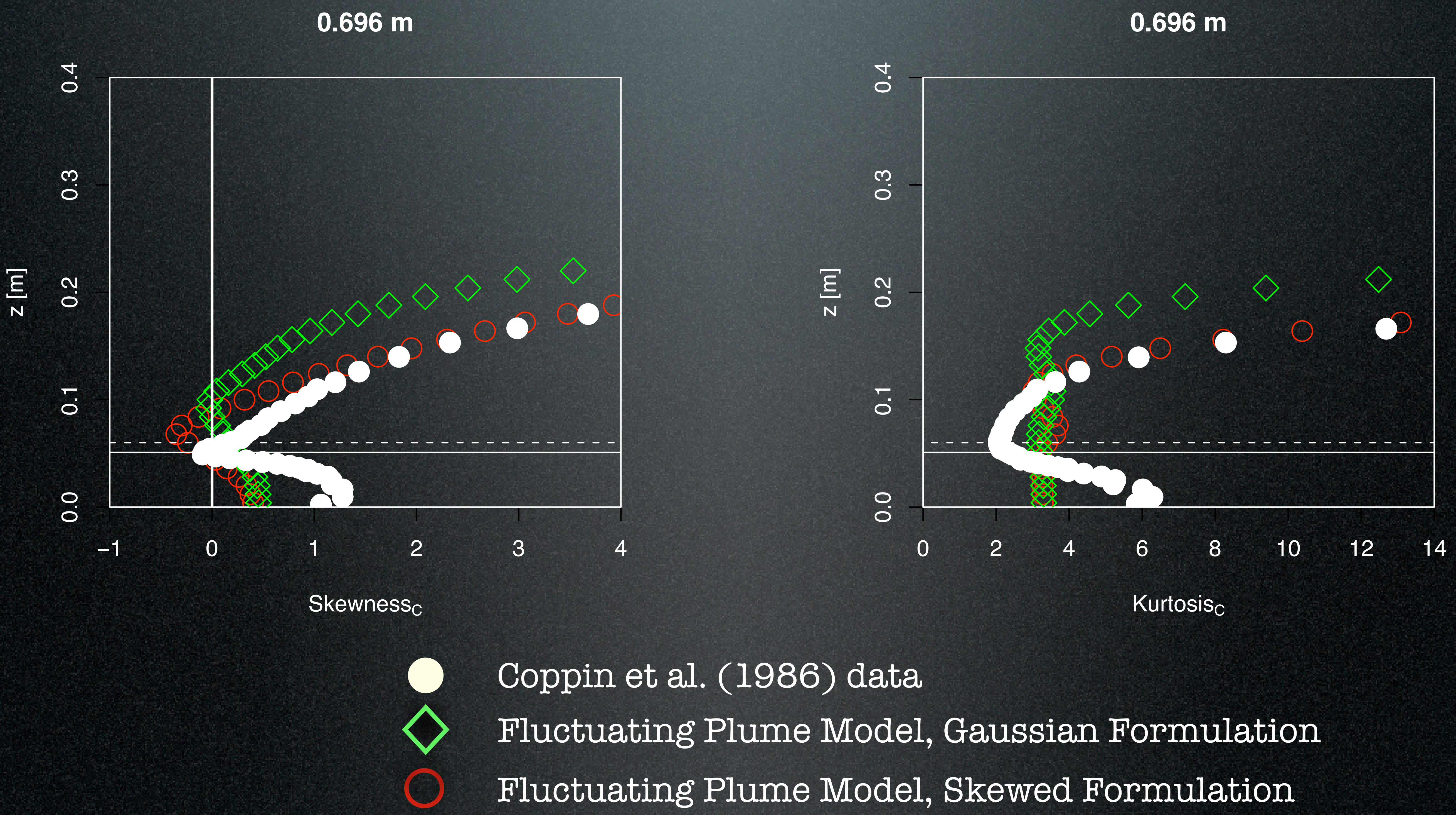
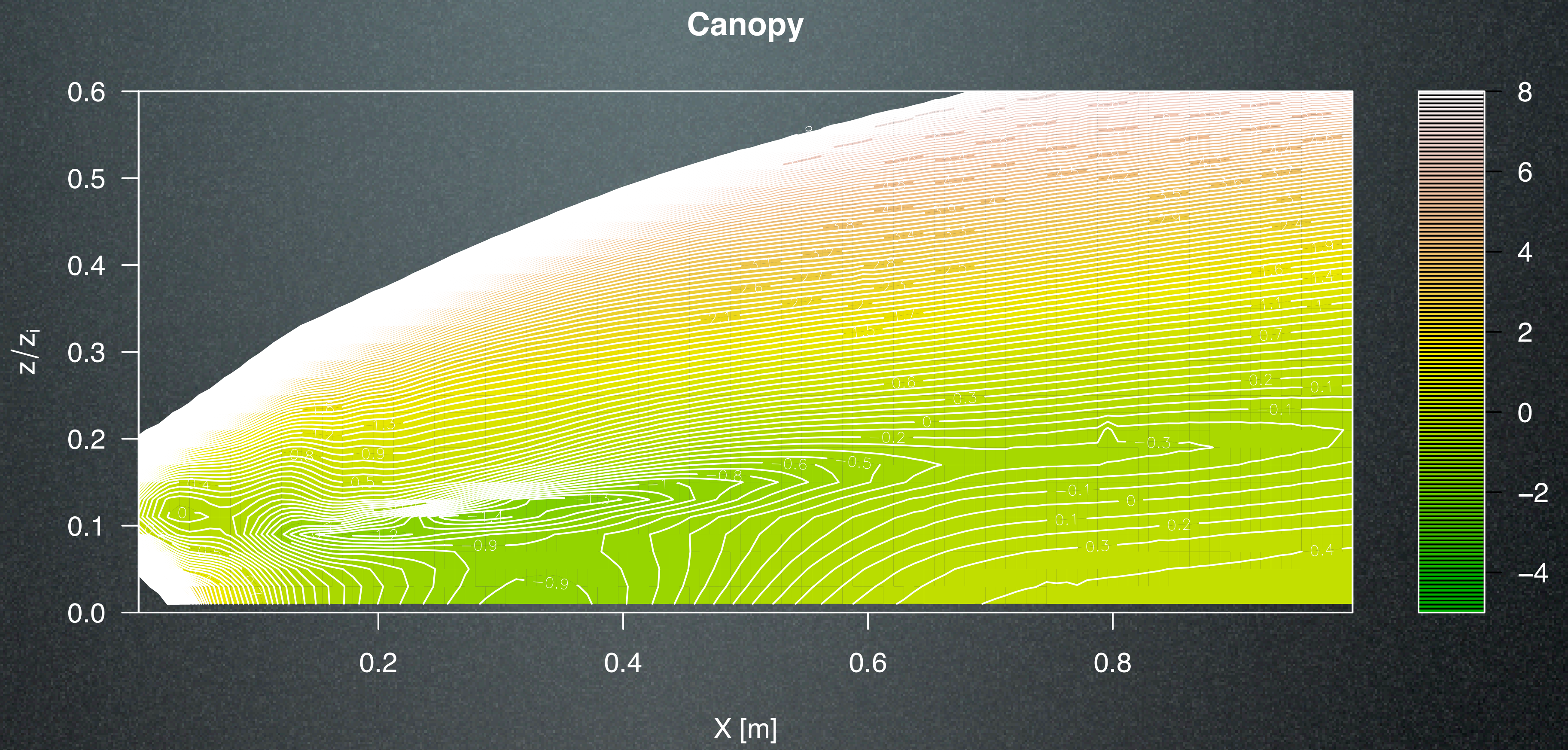
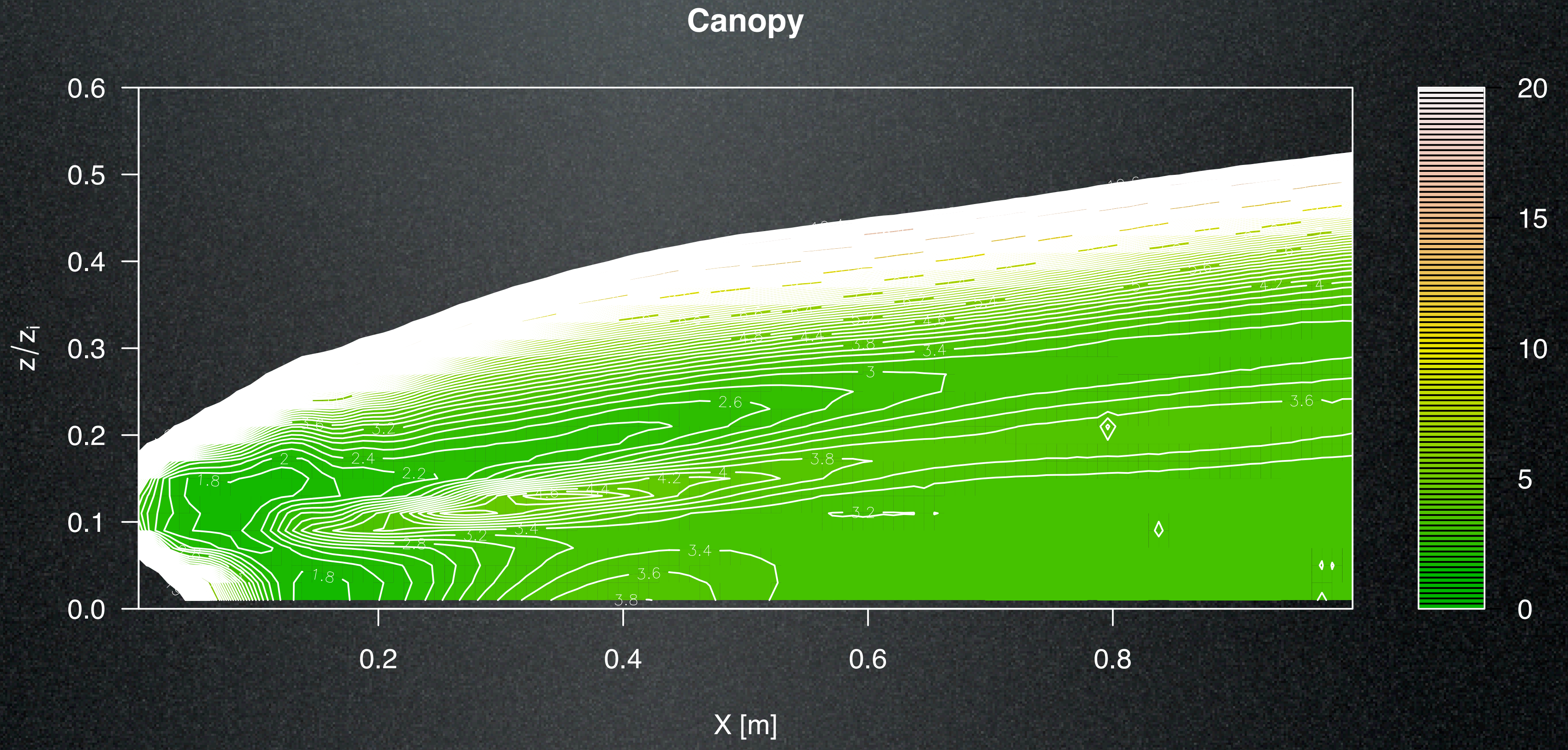


High Order Concentration Statistics- Canopy

Skewness



Kurtosis



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Need of datasets.

Thank you!

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